# TM5 Problem 3 – 24

For $\beta$ = 0.2 $s^{-1}$, produce computer plots like those shown in Figure 3-15 for a sinusoidal driven, damped oscillator where $x_p$*(t)*, $x_c$*(t)* and the sum *x(t)* are shown. Let *k = 1 kg/*$s^2$ and *m = 1 kg*. Do this for values of $\omega_D/\omega_S$ of *1/9*, *1/3*, *1.1*, *3* and *6*. For the $x_c$*(t)* solution (Eqn. 3.40, the underdamped case), let the phase angle $\delta$ *= 0*. And the amplitude *A = -1 m*. For the $x_p$*(t)* solution (Eqn. 3.60), let *A = 1 m/*$s^2$, but calculate $\delta$. What do you observe about the relative amplitudes of the two solutions as $\omega_D$ increases? Why does this occur? For $\omega_D/\omega_S$ *= 6*, let *A = 20 m/*$s^2$ for $x_p$*(t)* and produce the plot again.

In[1]:=
```
xc = A * Exp[-β * t] * Cos[ωS * t - θ]
xp = B * Cos[ωD * t - ϕ]
```

$$B = \frac{\frac{F_0}{m}}{\sqrt{\left(\omega_N{}^2 - \omega_D{}^2\right)^2 + 4\,\beta^2\,\omega_D{}^2}}$$

$$\phi = \text{ArcTan}\left[\frac{2 * \beta * \omega_D}{\omega_N{}^2 - \omega_D{}^2}\right]$$

Out[1]= $A\,e^{-t\beta}\,\text{Cos}[\theta - t\,\omega_S]$

Out[2]= $B\,\text{Cos}[\phi - t\,\omega_D]$

Out[3]= $$\frac{F_0}{m\sqrt{4\,\beta^2\,\omega_D^2 + \left(-\omega_D^2 + \omega_N^2\right)^2}}$$

Out[4]= $$\text{ArcTan}\left[\frac{2\,\beta\,\omega_D}{-\omega_D^2 + \omega_N^2}\right]$$

Giving

In[5]:=
```
x = xc + xp
```

Out[5]= $$A\,e^{-t\beta}\,\text{Cos}[\theta - t\,\omega_S] + \frac{\text{Cos}\left[\text{ArcTan}\left[\frac{2\,\beta\,\omega_D}{-\omega_D^2+\omega_N^2}\right] - t\,\omega_D\right]F_0}{m\sqrt{4\,\beta^2\,\omega_D^2 + \left(-\omega_D^2 + \omega_N^2\right)^2}}$$

Taking values of

```
In[6]:= θ = 0;
m = 1;
k = 1;
β = 0.2;
A = -1;
F0 = 1;
```

$$\omega_N = \sqrt{\frac{k}{m}};$$

$$\omega_S = \sqrt{\omega_N{}^2 - \beta^2}$$

Out[13]= 0.979796

Which gives $\omega_1 = 0.979796\ s^{-1}$. This gives

```
In[14]:= Expand[xc]
Expand[xp]
Expand[x]
```

Out[14]= $-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t]$

Out[15]= $\frac{\mathrm{Cos}\left[\mathrm{ArcTan}\left[\frac{0.4\,\omega_D}{1-\omega_D^2}\right]-t\,\omega_D\right]}{\sqrt{0.16\,\omega_D^2+\left(1-\omega_D^2\right)^2}}$

Out[16]= $-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t]+\frac{\mathrm{Cos}\left[\mathrm{ArcTan}\left[\frac{0.4\,\omega_D}{1-\omega_D^2}\right]-t\,\omega_D\right]}{\sqrt{0.16\,\omega_D^2+\left(1-\omega_D^2\right)^2}}$

To make the plots, first take $\frac{\omega_D}{\omega_S} = \frac{1}{9}$ so that $\omega_D = \frac{\omega_S}{9}$, or

In[17]:= $\omega_S$

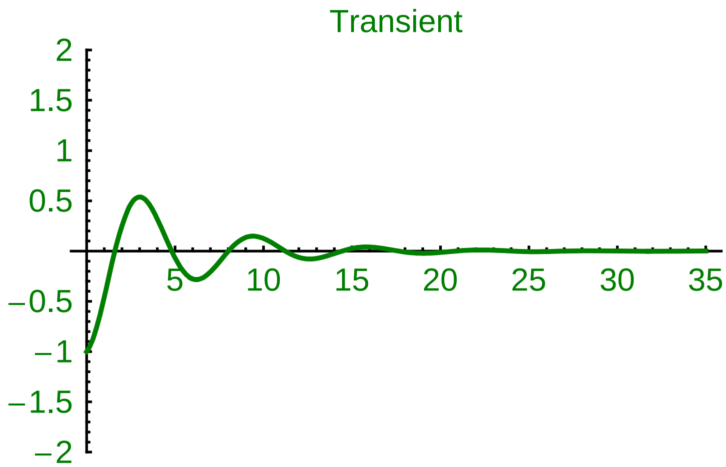
$$\omega_D = \frac{\omega_S}{9}$$

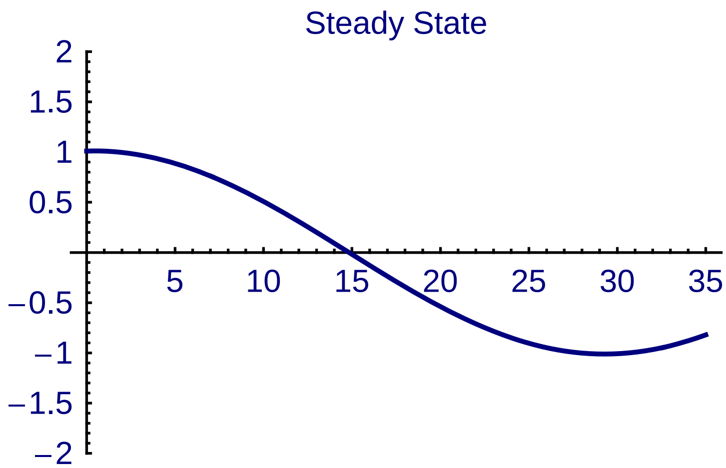
Out[17]= 0.979796

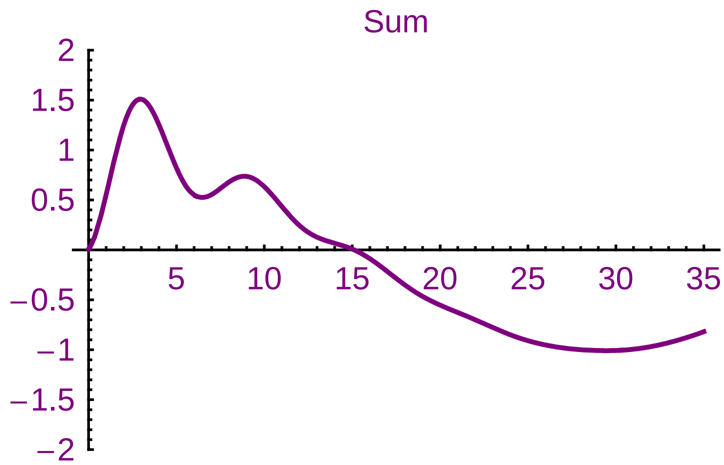
Out[18]= 0.108866

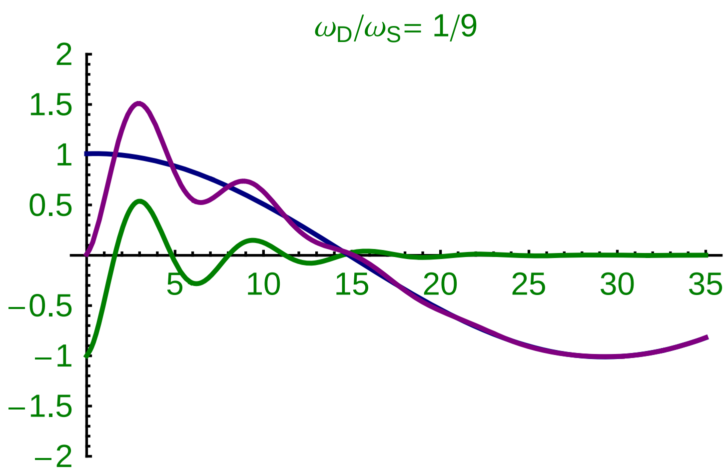
Plot for $\frac{\omega_D}{\omega_S} = \frac{1}{9}$

In[19]:=
```
pxc1 = Plot[xc, {t, 0, 35}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5`, 0]},
   PlotRange → {-2, 2}, PlotPoints → 100, PlotStyle →
    {{RGBColor[0, 0.5`, 0], Thickness[0.0075`]}}, PlotLabel → "Transient"]
pxp1 = Plot[xp, {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
     FontSize → 12, FontColor → RGBColor[0, 0, 0.5`]}, PlotRange → {-2, 2},
   PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5`], Thickness[0.0075`]}},
   PlotLabel → "Steady State"]
px1 = Plot[x, {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
     FontSize → 12, FontColor → RGBColor[0.5`, 0, 0.5`]},
   PlotRange → {-2, 2}, PlotPoints → 100, PlotStyle →
    {{RGBColor[0.5`, 0, 0.5`], Thickness[0.0075`]}}, PlotLabel → "Sum"]
Show[pxc1, pxp1, px1, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
     \(D\)]\)/\!\(\*SubscriptBox[\(ω\), \(S\)]\)= 1/9"]
```






Now take $\frac{\omega_D}{\omega_S} = \frac{1}{3}$ so that $\omega_D = \frac{\omega_S}{3}$, or

In[23]:= **ωS**

**$\omega_D = \frac{\omega_S}{3}$**

Out[23]= 0.979796

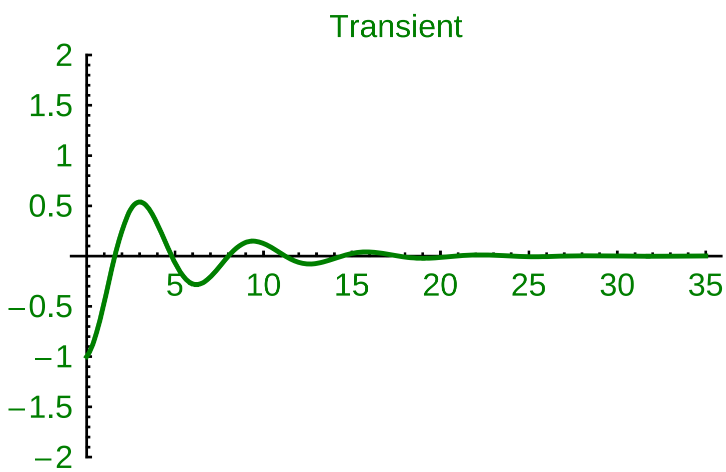
Out[24]= 0.326599

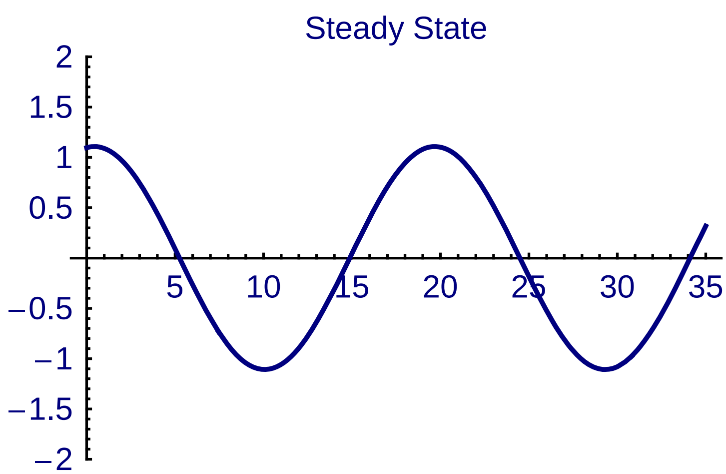
In[25]:= **x**

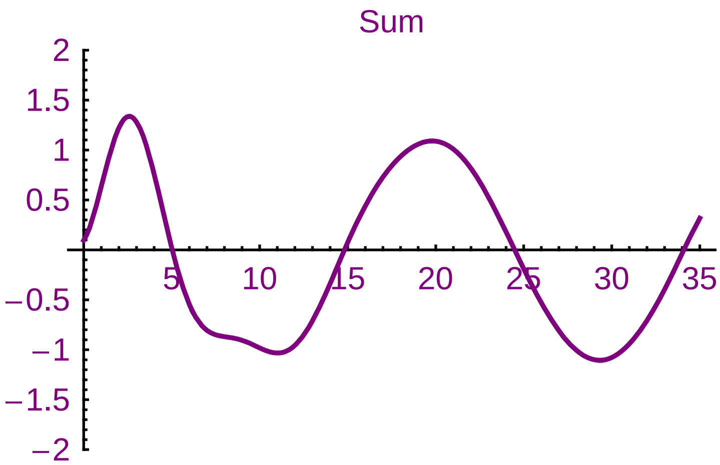
Out[25]= 1.10762 Cos[0.145209 - 0.326599 t] - $e^{-0.2 t}$ Cos[0.979796 t]

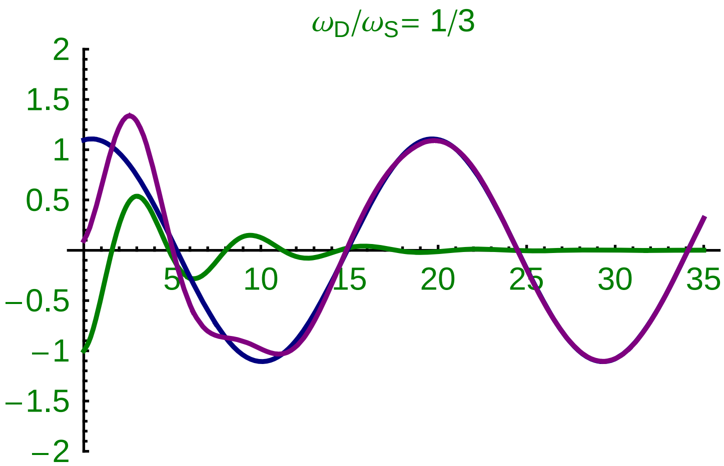
Plot for $\frac{\omega_D}{\omega_S} = \frac{1}{3}$

```
In[26]:= pxc2 = Plot[xc, {t, 0, 35}, BaseStyle →
            {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5`, 0]},
          PlotRange → {-2, 2}, PlotPoints → 100, PlotStyle →
            {{RGBColor[0, 0.5`, 0], Thickness[0.0075`]}}, PlotLabel → "Transient"]
        pxp2 = Plot[xp, {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
            FontSize → 12, FontColor → RGBColor[0, 0, 0.5`]}, PlotRange → {-2, 2},
          PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5`], Thickness[0.0075`]}},
          PlotLabel → "Steady State"]
        px2 = Plot[x, {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
            FontSize → 12, FontColor → RGBColor[0.5`, 0, 0.5`]},
          PlotRange → {-2, 2}, PlotPoints → 100, PlotStyle →
            {{RGBColor[0.5`, 0, 0.5`], Thickness[0.0075`]}}, PlotLabel → "Sum"]
        Show[pxc2, pxp2, px2, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
            \(D\)]\)/\!\(\*SubscriptBox[\(ω\), \(S\)]\)= 1/3"]
```





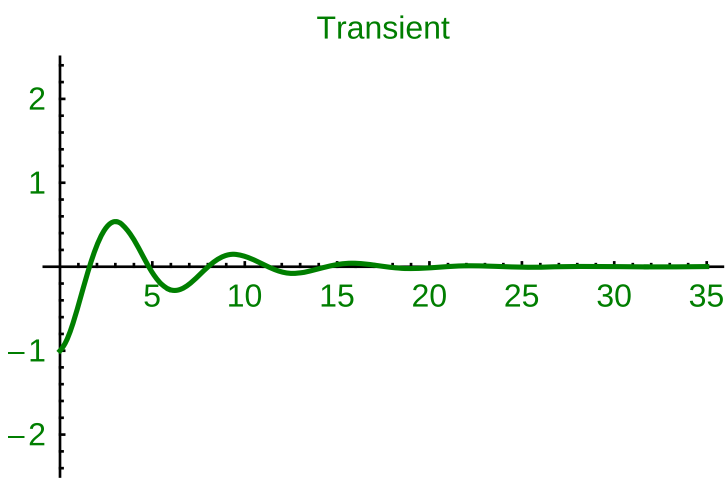



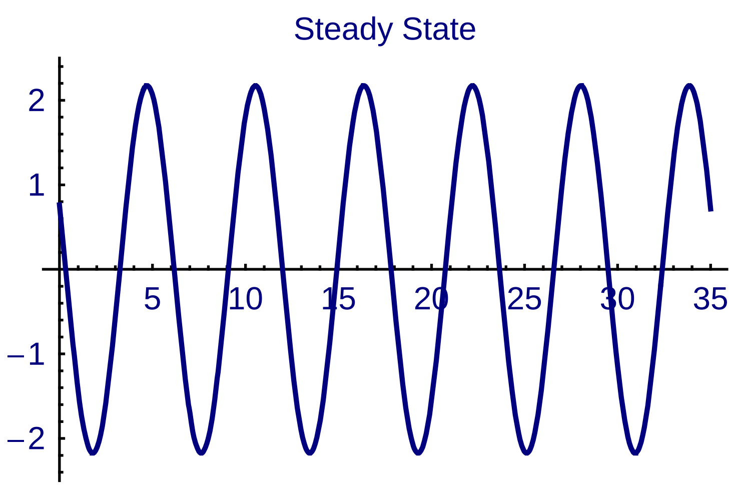

Next take $\frac{\omega_D}{\omega_S}$ = 1.1 so that $\omega_D$ = 1.1 $\omega_S$, or

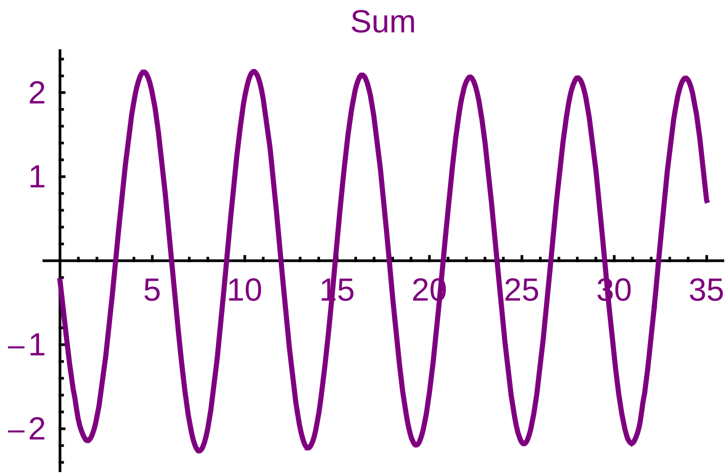
```
In[30]:= ωS
         ωD = 1.1 * ωS
Out[30]= 0.979796
Out[31]= 1.07778
In[32]:= x
Out[32]= -e^(-0.2 t) Cos[0.979796 t] + 2.17201 Cos[1.21216 + 1.07778 t]
```

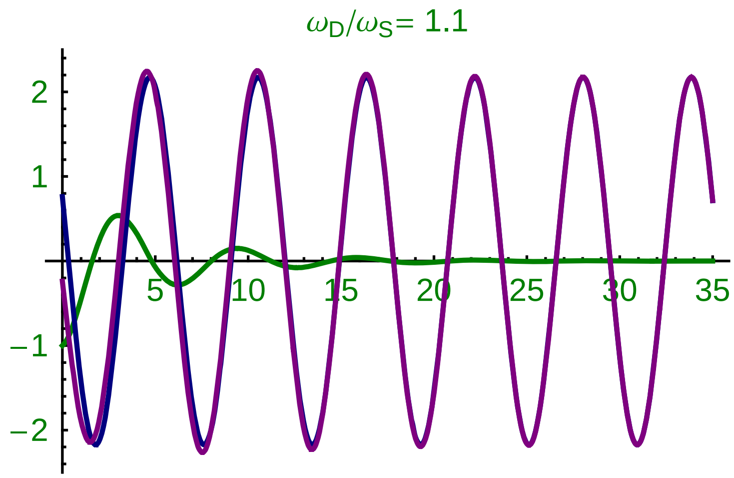
Plot for $\frac{\omega_D}{\omega_S}$ = 1.1

In[33]:=
```
pxc3 = Plot[xc, {t, 0, 35}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5`, 0]},
   PlotRange → {-2.5`, 2.5`}, PlotPoints → 100, PlotStyle →
    {{RGBColor[0, 0.5`, 0], Thickness[0.0075`]}}, PlotLabel → "Transient"]
pxp3 = Plot[xp, {t, 0, 35}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0, 0.5`]},
   PlotRange → {-2.5`, 2.5`}, PlotPoints → 100,
   PlotStyle → {{RGBColor[0, 0, 0.5`], Thickness[0.0075`]}},
   PlotLabel → "Steady State"]
px3 = Plot[x, {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
     FontSize → 12, FontColor → RGBColor[0.5`, 0, 0.5`]},
   PlotRange → {-2.5`, 2.5`}, PlotPoints → 100, PlotStyle →
    {{RGBColor[0.5`, 0, 0.5`], Thickness[0.0075`]}}, PlotLabel → "Sum"]
Show[pxc3, pxp3, px3, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
     \(D\)]\)/\!\(\*SubscriptBox[\(ω\), \(S\)]\)= 1.1"]
```






Next take $\frac{\omega_D}{\omega_S} = 3$ so that $\omega_D = 3\,\omega_S$, or

```
In[37]:= ωS
         ωD = 3 * ωS
Out[37]= 0.979796
Out[38]= 2.93939
```

```
In[39]:= x
Out[39]= -e^(-0.2 t) Cos[0.979796 t] + 0.129367 Cos[0.152697 + 2.93939 t]
```

$$-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t] + 0.129367\,\mathrm{Cos}[0.152697 + 2.93939\,t]$$

Plot for $\frac{\omega_D}{\omega_S} = 3$

```
In[40]:= pxc4 = Plot[xc, {t, 0, 35}, BaseStyle →
             {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5`, 0]},
           PlotRange → {-1, 1}, PlotPoints → 100, PlotStyle →
             {{RGBColor[0, 0.5`, 0], Thickness[0.0075`]}}, PlotLabel → "Transient"]
         pxp4 = Plot[xp, {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
              FontSize → 12, FontColor → RGBColor[0, 0, 0.5`]}, PlotRange → {-1, 1},
           PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5`], Thickness[0.0075`]}},
           PlotLabel → "Steady State"]
         px4 = Plot[x, {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
              FontSize → 12, FontColor → RGBColor[0.5`, 0, 0.5`]},
           PlotRange → {-1, 1}, PlotPoints → 100, PlotStyle →
             {{RGBColor[0.5`, 0, 0.5`], Thickness[0.0075`]}}, PlotLabel → "Sum"]
         Show[pxc4, pxp4, px4, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
              \(D\)]\)/\!\(\*SubscriptBox[\(ω\), \(S\)]\)= 3"]
```